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INERTIAL MIGRATION BASED CONCENTRATION FACTORS FOR SUSPENSIONS OF CHLORELLA MICRO-ALGAE IN BRANCHED TUBES

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ABSTRACT When a dilute suspension flows in the laminar regime through a tube, under certain conditions the suspended particles migrate radially to an equilibrium radial position. Branched tubes can use this radial concentration distribution to concentrate dilute suspensions. Suspensions of microalgae, Chlorella vulgaris, were pumped through tubes of various diameters for tube Reynolds number ranging from 47–1839 and photographed. Upstream particle concentration profiles were obtained by image analysis of the photographs. The dividing stream surfaces in branched tubes were obtained from the three-dimensional numerical solutions of the Navier-Stokes equations for steady, laminar, and homogeneous flow through tubes having one and two orthogonal branches. Concentration factors for Chlorella suspensions in branched tubes, predicted by a general method, fall in the range of 1.0–1.3

Introduction

Segré and Silberberg (1962) showed that when a dilute suspension of neutrally buoyant rigid spheres is transported along in Poiseuille flow, the particles are subjected to lateral forces that result in migration to an equilibrium radial region located approximately 60% of the distance from the tube axis to the tube wall. This phenomenon is referred to as inertial migration because Bretherton (1962) demonstrated that if the inertial terms in the equations of motion of a fluid are neglected, then no lateral (the direction perpendicular to the dominant flow direction) force can exist on a body of revolution in a non-uniform, unidirectional flow. The driving forces behind inertial migration originate from the specific pressure and shear stress distribution on the surface of a particle moving in a non-uniform flow field. The mechanisms causing such

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a force distribution have been identified as inertial lift due to the presence of shear, wall repulsion due to the lubrication effect, lift due to particle rotation and, in the case of Poiseuille flow, a lift due to the added presence of velocity profile curvature (Feng *et al.*, 1994).

The phenomenon of inertial migration has also been observed for nonspherical particles. Rakow and Chappell (1987) found that a suspension of helical shaped microalgae, Spirulina platensis, flowing through a 0.65 mm diameter tube, migrated radially to a relatively narrow annular region. As the tube Reynolds number was increased within the laminar regime, the particles (microalgae) concentrated within an increasingly narrow region concentric with the tube axis. Based on an order of magnitude analysis involving a balance between viscous and inertial forces at equilibrium, Rakow *et al.* (1989) found that the diameter of the concentrated particle region varied inversely with the square root of the tube Reynolds number. They were able to fit the experimental observations of Rakow and Chappell (1987) to an equation of the form

$$\frac{d}{D} = K\,Re^{-1/2} + m \quad ,$$

where d is the diameter of the concentrated particle region, D is the tube diameter, Re is the tube Reynolds number, and K and m are parameters that may depend on both particle size and concentration. Parameters affecting inertial migration of particles in Poiseuille flow include particle to tube size ratio, the tube Reynolds number, Re, the concentration of the suspension, the shape of the particle, the deformability of the particle, the density difference between the particle and fluid, and whether or not the particle undergoes rotation.

Tubes with branches were first used (Rakow, 1991) as a separation device by Rakow and Fernald (1991) for concentrating Spirulina microalgae suspensions. Concentration factors as high as 3 were obtained for laminar flows of Spirulina suspensions through branched tubes. The concentration factor is defined as the ratio of the exit particle concentration to that of the inlet concentration into the main tube of the separator. Branched tube separation devices can be used in a continuous mode, and by employing a staged system, where each stage is made up of a set of branched tubes, it is possible to achieve high recovery and large concentration factors (Poflee *et al.*, 1994). In their experiments on concentrating Spirulina suspensions using branched tubes, Ryan *et al.* (1996), showed that by using seven stages it was possible to increase the concentration by a factor of 33, starting from a feed concentration of 0.015% *DM* (percent dry mass). Moreover, they obtained an overall recovery of 91.5%.

Efficient skimming of the flow field in branched tubes may be obtained by adjusting the flow fraction through the branch such that the dividing stream surface is located within the dilute region of the flow field as the suspension arrives at the branch junction. The dividing stream surface is defined as the collection of streamlines acting as the boundary between the portions of the flow entering each downstream branch (Rong and Carr, 1990). Figure 1 shows a schematic representation of the dividing streamline in branched tube flows. If the suspension is dilute and the particles are sufficiently small, and after the particles reach their radial equilibrium positions in the tube it is assumed that the particles follow the flow streamline, then it may be reasonable to use the flow structure determined for a single phase fluid to predict the concentration factors in branched tubes. Assuming that rigid, neutrally buoyant particles

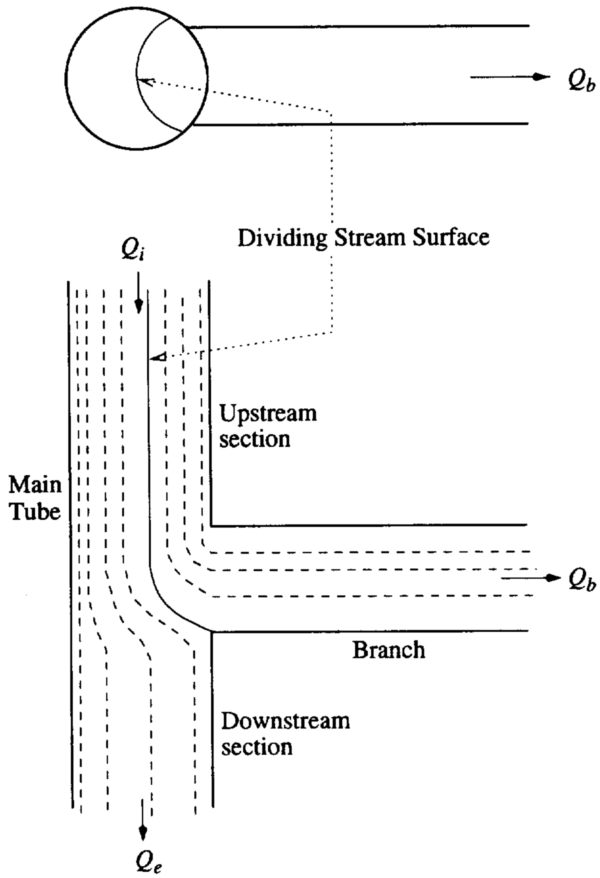

Fig. 1. Schematic of flow splitting in branched tube.

generally follow the streamlines that would exist in a single phase viscous fluid under the same flow conditions, it is possible to evaluate the flow skimming in branched tubes provided the particle concentration profiles are also known. Integrating the velocity field in combination with the concentration profile over the region demarcated by the dividing stream surface near the junction, it is possible to predict the amount of material flowing into the branch. As a first step, in this work we have studied, numerically, the flow field for a single-phase, Newtonian, incompressible fluid flowing through branched tubes with orthogonal branches. T-type junctions may have more than one side branch, such as those used by Rakow and Fernald (1991) for concentrating microalgae suspensions, which consisted of two opposing 90° side branches on the main tube. Such a two-branch system provides a symmetrical skimming of the flow field in the main tube and thus may be advantageous from the point of view of using branched tubes for suspension concentration.

The primary parameters governing the shape and location of the dividing stream surface are the diameter ratio of the branch tube to the main tube, γ, the upstream main tube Reynolds number, *Re*, and the fraction of the total inlet flow rate leaving through the branch, Q_f (Enden and Popel, 1992; Carr and Kotha, 1995). In this work, *Re* is defined as $D_m \langle V_{m,i} \rangle \rho / \mu$, where D_m is the diameter of the main tube, $\langle V_{m,i} \rangle$ is the average velocity at the inlet of the main tube, and ρ and μ are the density and the viscosity of the fluid, respectively. Several experimental studies have been done regarding the determination of dividing stream surface in branched tube flows (Pinchak and Ostrach, 1976; Ofjord and Clausen, 1983; Rong and Carr, 1990; Carr and Kotha, 1995). For $Re = 194$, Carr and Kotha (1995) observed closed dividing stream surfaces for $Q_f \geq 0.5$. This observation indicates that fluid close to the tube wall, and around the circumference of the main tube at the junction, is being drawn into the branch while the higher momentum fluid at the core of the main tube continues its motion downstream in the main tube. Three-dimensional flow in a straight main tube with a very small radius side branch ($\gamma << 1$) at a bifurcation angle of 90° was studied numerically by Tutty (1988). Yan *et al.* (1991) solved the problem formulated by Tutty (1988) using asymptotic analysis, with the primary aim of determining the three-dimensional shape of the capture tube, that is, the entrainment region. Enden and Popel (1992) determined numerically the dividing stream surfaces in a single-branch T-type bifurcation for $Re \leq 2.5$ by solving the steady state three-dimensional Navier-Stokes equations using a finite element method. Currently, for T-type bifurcations with two orthogonal branches, no experimental or three-dimensional numerical results are available in the literature regarding dividing stream surfaces or secondary flows at the junction.

The present study will examine the inertial migration-based concentration of suspensions of Chlorella vulgaris in branched tubes. Chlorella vulgaris is roughly spherical in shape and is approximately 5 µm in size. Its density is close to that of water, so that the suspension is nearly neutrally buoyant. This work is divided into two parts. The first part is concerned with the numerical determination of the location and shape of the dividing stream surfaces in branched tubes having one or two orthogonal side branches and the effect of parameters, upstream tube Reynolds number, Re, the diameter ratio, γ, and the flow fraction, Q_f, on the dividing stream surface. The dividing stream surfaces have been determined from the three-dimensional flow field in the T-type branched tubes for the flow of a homogeneous Newtonian fluid. The flow field in branched tubes was determined by obtaining numerical solutions of the three-dimensional Navier-Stokes equations.

The second part of the study is concerned with the determination of the inertial migration characteristics of Chlorella vulgaris microalgae in tube flow and its separation in branched tubes. The inertial migration characteristics of Chlorella were determined by taking photographs of suspensions flowing through tubes of various diameters and for various tube Reynolds numbers. The particle concentration profiles in conjunction with the flow field in branched tubes were then used to calculate the separation that may be achieved in branched tubes for various flow conditions.

Dividing stream surfaces in branched tubes

The equations describing the motion of the fluid in a branched tube are the continuity and the Navier-Stokes equations. It is assumed that the fluid is incompressible and Newtonian, so that the density, ρ, and the viscosity, μ, are constant. The equations are non-dimensionalized using the diameter of the main tube, D_m, as the characteristic length, the average flow velocity at the inlet of the main tube, $\langle V_{m,i} \rangle$, as the characteristic velocity, and $\rho \langle V_{m,i} \rangle^2$ as the characteristic pressure scale. The dimensionless forms of the governing equations, written in integral form, are then

$$(1) \qquad \iint_s \mathbf{u} \cdot d\mathbf{A} = 0 \quad ,$$

and

$$\frac{\partial}{\partial t}\iiint_v \mathbf{u}dV + \iiint_v \mathbf{u}\cdot\nabla\mathbf{u}dV = -\iint_s p\,d\mathbf{A} + \frac{1}{Re}\iint_s\left(\nabla\mathbf{u} + \nabla\mathbf{u}^T\right)\cdot d\mathbf{A} \quad , \qquad (2)$$

where the velocity vector is $\mathbf{u} = (u_x, u_y, u_z)$, corresponding to the Cartesian coordinates (x, y, z), p is the dynamic pressure, and the Reynolds number is defined as $Re = D_m \langle V_{m,i} \rangle \rho/\mu$. The quantity **A** represents the area vector of the surface of the control volume. Equation *(2)* is expressed with the time derivative outside the integral sign because the coordinate grid is invariant with respect to time in this problem. Although steady state solutions are sought, Eq. *(2)* is written in a time-dependent form because of the specific solution algorithm used in this work. A cell-centered finite-volume formulation has been used to discretize the governing Eqs. *(1)* and *(2)*. The details of the finite-volume formulation are described by Dandy and Dwyer (1990). A composite grid generation scheme was used to map the branched tube domain.

In an actual branched tube system used for separation, it is expected that the upstream and downstream lengths of the main tube, and those of the branches, are sufficiently long, leading eventually to the formation of fully developed flows there. Therefore, at the inlet and exit of the main tube, fully developed Poiseuille velocity profiles are imposed. The exit boundary condition for the branch comes from the fully developed Poiseuille velocity profile corresponding to the fraction of inlet flow leaving through the branch. The pressure boundary conditions at the exits of the main tube and branch come from the application of linear pressure drops, corresponding to a fully developed Poiseuille flow at those locations. At the tube walls the no-slip boundary condition is imposed on velocity and zero normal derivative on pressure. Further details on the solution procedure and domain discretization can be found in the work of Poflee (1996). Each computation is carried out for specified values of the three parameters: Re, γ, and Q_f.

The flow field obtained by the numerical solution of the Navier-Stokes equations in T-type branched tubes is analyzed to obtain the dividing stream surface (branch influence zone). The dividing stream surfaces are mapped using the procedure described in Enden and Popel (1992). The effect of diameter ratio on the dividing stream surface in single-branch tubes is shown in Fig. 2. See Fig. 4(a) for the axis nomenclature, which shows a view of the bifurcation system as seen from the inlet of the main tube looking axially downstream. Increasing the diameter ratio of the branch has the effect of providing a larger area/opening along the circumference of the main tube for the fluid to flow into the branch. Consequently, the influence zone and the dividing stream surface run closer to, and along the wall of the main tube for larger γ. Using a dye filament apparatus, Rong and Carr (1990) determined the dividing stream surface locations in T-type bifurcations having one branch for Re close to unity. Figure 3 compares the experimentally determined dividing stream surface by Rong and Carr (1990) with that determined computationally in this work for $Re = 0.86$, $Q_f = 0.2$, and $\gamma = 0.5$.

Results on dividing stream surfaces in a T-type bifurcation with two orthogonal branches are presented next. The Reynolds number range studied for this system was $10 \le Re \le 250$. Since flow is symmetric with respect to the perpendicular plane in a T-type bifurcation with two branches, only the right half of the domain shown in Fig. 4(a) is used for the computation. Figure 4(b) illustrates the effect of flow fraction on dividing stream surface location. The branch influence zone becomes larger with increasing flow fraction through the branch. Figure 5 illustrates the effect of Reynolds number on the dividing

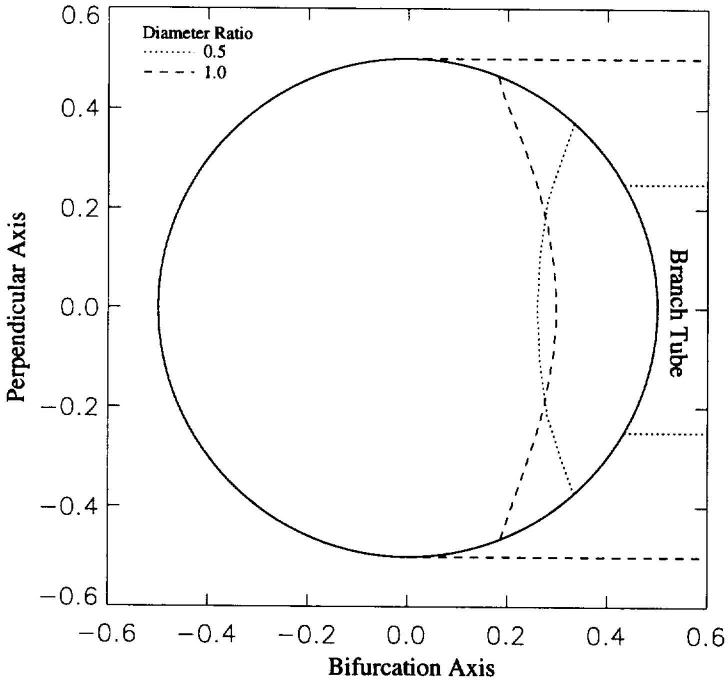

Fig. 2. The effect of diameter ratio of branch to main on the dividing stream surface for $Re = 13$ and $Q_f = 0.1$ in a T-type bifurcation with one branch.

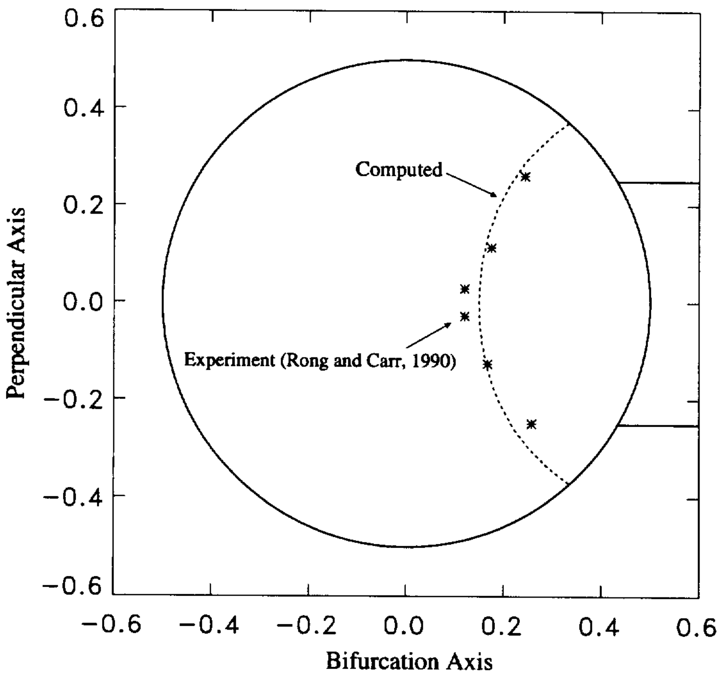

Fig. 3. Comparison of experimental data with the numerical results computed in this work; $Re = 0.86$, $Q_f = 0.2$, and $\gamma = 0.5$.

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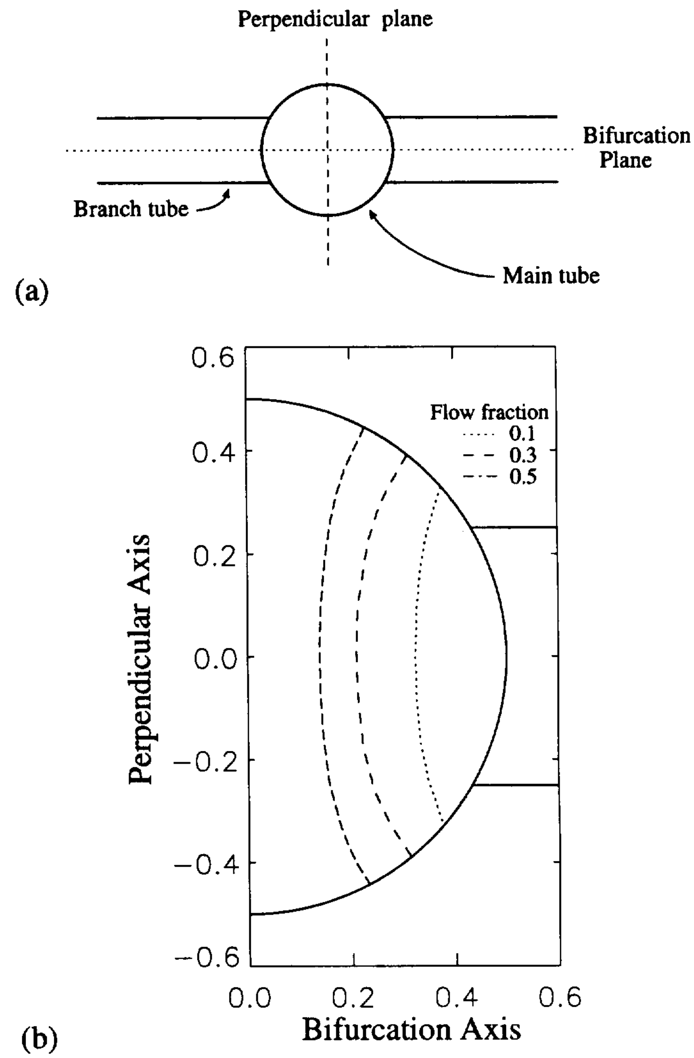


Fig. 4. (a) Illustration of the bifurcation and perpendicular planes of branched tubes, and (b) the effect of Q_f on the dividing stream surface in a T-type bifurcation with two branches for $\gamma = 0.5$ and $Re = 10$.

stream surface. The shape of the dividing stream surface changes from concave to convex with increasing *Re*. Moreover, for $Re = 150$ and 250, the dividing stream surface does not intersect the tube wall, it will meet with its symmetrical half about the perpendicular plane to form a closed, elliptical shaped dividing stream surface (see Fig. 6). Such dividing stream surfaces have been observed by Carr and Kotha (1995) for $Re = 194$ in a T-type bifurcation with one orthogonal branch. This observation indicates that the low momentum fluid close to the tube wall and around the circumference of the main tube at the junction is being drawn into the branch; while the higher momentum fluid at the core of the main tube continues its motion downstream in the main tube.

In Poflee *et al.* (1994) regarding separation of Spirulina suspensions in branched tubes at high Reynolds number ($1200 \leq Re \leq 1800$), closed circular dividing stream surfaces were assumed. Moreover, the radius of this circular

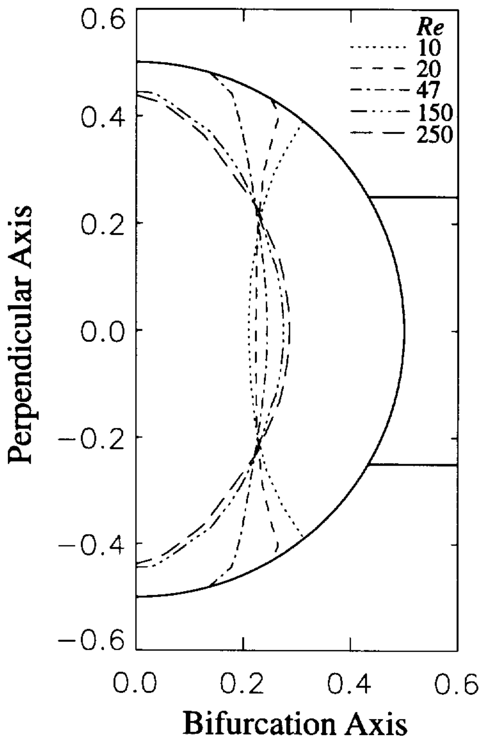
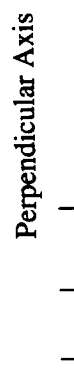


Fig. 5. The effect of *Re* on the dividing stream surface in a T-type bifurcation with two branches for $\gamma = 0.5$ and $Q_f = 0.3$.

dividing stream surface for a given flow fraction through the branch was determined based on mass balance, and the assumption of Poiseuille velocity profile at the junction. Based on this assumption, the radius of the ideal circular dividing stream surface for a given flow fraction Q_f through the branches is given by

$$k = \sqrt{\left(1 - \sqrt{Q_f}\right)} \tag{3}$$

where $k = a/R$, R is the radius of the main tube and a is the radius of the circular dividing stream surface. In order to compare the ideal circular dividing stream surface with the closed dividing stream surface for $Re = 250$, $Q_f = 0.3$, $\gamma = 0.5$; the two dividing stream surfaces are plotted in Fig. 6. Note that the numerically computed dividing stream surface has an elliptic shape with its center concentric with the tube center. From this result, it is arguable that a circular dividing stream surface is a reasonably good approximation of the actual dividing stream surface for $Re > 250$. However, the tendency to produce secondary flows also increases with Reynolds number, and its effect on the dividing stream surface at those high Reynolds numbers remains to be determined. The experimental procedure for determining the inertial migration characteristics of Chlorella suspensions is presented next and is followed by the description of the procedure used to compute the separation in branched tubes.



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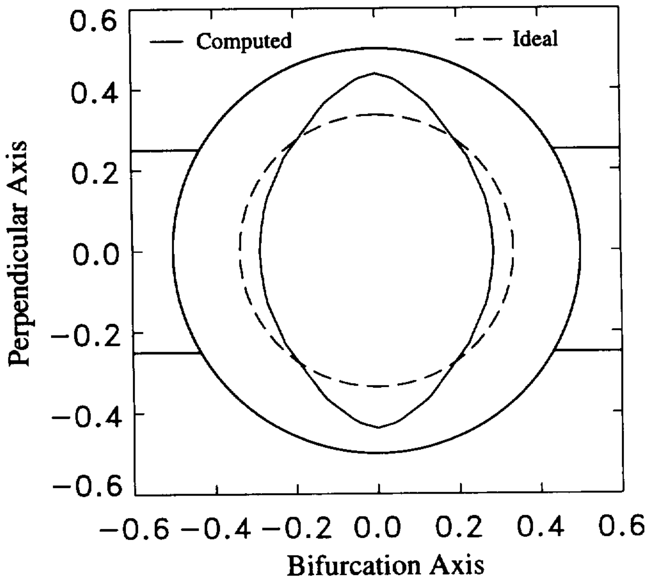

Fig. 6. Comparison of the numerically computed dividing stream surface with the ideal circular dividing stream surface for $Re = 250$, $Q_f = 0.3$, and $\gamma = 0.5$, in a T-type bifurcation with two branches.

Inertial migration of Chlorella microalgae in tube flow

Experimental method

A constantly stirred suspension of Chlorella vulgaris was pumped through a vertically mounted glass tube and photographed. The suspension was pumped through two different diameter tubes, 203 μm and 405 μm. The tube Reynolds number studied ranged from 47–1839. Chlorella vulgaris is approximately 5 μm in size and is roughly spherical in shape. The concentration of the culture pumped through the tubes was approximately 0.03% *DM*. A concentration value of 0.03% *DM* is very dilute, as is usually the case for microalgae suspensions grown in bioreactors. The experimental method is similar to that used in Rakow *et al.* (1989) to study the inertial migration characteristics of Spirulina platensis suspensions, with the following exceptions:

1. The flow rate measurement was accomplished by connecting the output end of the tube to the input of a Curtin automatic burette. A glass valve was attached to the output which was routed back to the reservoir (Erlenmeyer flask). With the output valve closed, the suspension was collected in the burette; then, after the measurement was made, the valve was opened and the suspension flowed back to the reservoir.

2. Various speeds of Kodak T-Max film were used and developed with T-Max developer.

3. The Chlorella vulgaris cultures (approximately 200 ml of suspension) were grown in 'Alagaro' medium obtained from Carolina Biological Supply (Burlington, N.C.), as were the seed cultures. The culture was grown in 500 ml Erlenmeyer flasks with periodic agitation by hand (2 or 3 times a week). Illumination and temperature were ambient in the laboratory. The age of the cultures varied, but none was old enough to

show significant flocculation. This was substantiated by hemacytometer photos taken at various points in the experiments. Dry mass concentration was also determined from hemacytometer counts.

4. A combination of three Masterflex Peristaltic Pumps with different tube sizes was used in order to expand the range of Reynolds numbers obtainable.

The photographs of the flowing suspension were digitized by recapturing them with a Bosch video camera (Chalnicon tube) equipped with a macro-lens with 256 gray levels through a professional vision software package (Visilog by Noesis, Velizy-Villacoublay, France) in such a way that the tube axis lies along a vertical line of the image. Further details are provided in Poflee (1996).

Procedure for computing concentration factors in branched tubes

Information on the dividing stream surface and the concentration profile at an upstream location, sufficiently far from the junction, may be used with the Poiseuille velocity profile to evaluate the fraction of particles going into the branch. Let the radius of the main tube, R_m, be the characteristic length, and the average velocity at the inlet of the main tube, $\langle V_{m,i}\rangle$, be the characteristic velocity. The expression inlet of the main tube refers to an upstream location in the main tube where the Poiseuille velocity profile is fully established and the effect of the branch junction is not felt. From an analysis of the numerical solution of the flow field in two-branch tubes it has been found that the branch junction disturbs the flow over a distance less than half the tube diameter upstream of the junction for the Reynolds number ranging from 10–250 (Poflee, 1996). Moreover, it was observed that the affected upstream lengths decrease with increasing tube Reynolds number. The suspension concentration may be measured as the number of particles per unit volume of suspension, or the fraction of volume occupied by particles, or the mass of particles per unit volume of suspension. Let the average concentration of particles at the inlet of the main tube, $\langle c^*_{m,i}\rangle$, be the characteristic concentration, and it is defined as

$$(4)\qquad \left\langle c^*_{m,i}\right\rangle = \frac{\int_0^{R^*_m} c^* u^*_z 2\pi\, r^*\, dr^*}{\int_0^{R^*_m} u^*_z 2\pi\, r^*\, dr^*}\quad ,$$

where u_z is the axial velocity at a radial position r, and the * is used to indicate that the quantity is dimensional.

The dimensionless rate at which particles enter the inlet of the main tube can be computed by:

$$(5)\qquad S_{m,i} = \int_0^1 c u_z 2\pi\, r\, dr\quad ,$$

where $r = r^*/R^*_m$ is the dimensionless radial position, $c = c^*/\langle c^*_{m,i}\rangle$, is the dimensionless concentration, and $u_z = u^*_z/\langle V^*_{m,i}\rangle$, is the dimensionless axial velocity. Henceforth, all the quantities will be dimensionless unless otherwise specified. The fraction of particles, f_ψ, going through the influence zone, ψ, demarcated by the dividing stream surface at the same upstream location (see Fig. 7) is then given by

$$(6)\qquad f_\psi = \frac{\int_\psi c\, u_z\, d\psi}{S_i}\quad .$$

Based on the assumptions stated at the beginning of this section, f_ψ also represents the fraction of material leaving the branch. Let n be the number of branches equally sharing the flow from the parent tube. Then the average concentration of particles leaving a branch is given by

$$\langle c_b \rangle = \frac{f_\psi \, S_{m,i}}{n Q_b} \quad , \tag{7}$$

where Q_b is the volumetric flow rate through one branch. If Q_f is the fraction of the total inlet flow rate leaving through the branches, which is being equally shared by the n branches, then

$$Q_b = \frac{Q_f \, Q_{m,i}}{n} \quad , \tag{8}$$

where $Q_{m,i}$ is the total inlet flow rate in the main tube. Substituting for Q_b in Eq. *(7)* gives

$$\langle c_b \rangle = \frac{f_\psi \, S_{m,i}}{Q_f \, Q_{m,i}} \quad . \tag{9}$$

However, the ratio $S_{m,i}/Q_{m,i}$ is the dimensionless average inlet concentration in the main tube and is therefore equal to unity because the characteristic concentration dimension is the inlet concentration in the main tube. Thus, Eq. *(9)* reduces to

$$\langle c_b \rangle = \frac{f_\psi}{Q_f} \quad . \tag{10}$$

Similarly, the average concentration of particles leaving the exit of the main tube, $\langle c_{m,e} \rangle$, can be obtained by

$$\langle c_{m,e} \rangle = \frac{1 - f_\psi}{1 - Q_f} \quad . \tag{11}$$

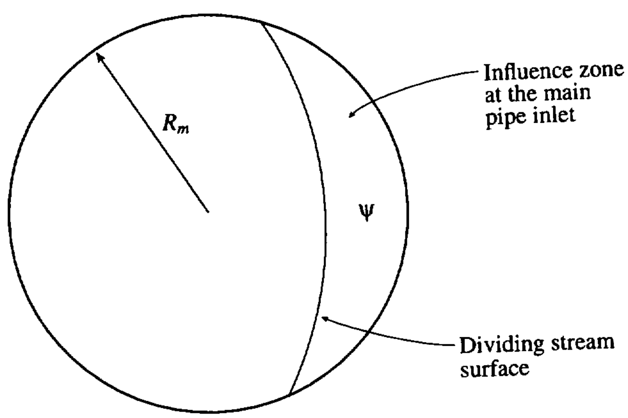

Fig. 7. Schematic for the evaluation of the flow through the influence zone at an upstream location in the main tube.

Moreover, if concentration factor is defined as the ratio of the average concentration of particles leaving a tube to the average concentration at the inlet of the main tube, then $\langle c_b \rangle$ and $\langle c_{m,e} \rangle$ also represent the concentration factors obtained at the exits of the branch and the main tube, respectively.

Results for inertial migration of Chlorella in tube flows

Figure 8 shows the upstream particle concentration profiles of Chlorella suspension obtained from image analysis of the photographs of suspension flowing through a tube of diameter 203 μm at $Re = 47$. The concentration is expressed in terms of gray scale obtained form the digitized images. In order to determine concentration factors, which is the final objective, relative units of concentration are sufficient. Note the peak concentration at a radial position of approximately 0.6 in Fig. 8. This result is in agreement with Segré and Silberberg's (1962) original results for spheres at low *Re*, where they found the peak concentration at approximately the same radial position. Figure 9 shows the upstream particle concentration profile of Chlorella vulgaris suspensions for the higher Reynolds numbers. The suspension was pumped through a 405 μm diameter tube. Note the cell depleted region around the tube axis as compared to that in the earlier case with the lower Reynolds number in Fig. 8.

The concentration profiles were used in conjunction with the computed dividing stream surfaces in branched tubes to obtain the concentration factors using Eqs. *(10)* and *(11)*. For $Re = 47$, the results are shown in Table 1 for various flow fractions, Q_f, and branch to main tube diameter ratios, γ. The estimated concentration factors at the exit of the main tube for Chlorella suspension at $Re = 47$ (see Table 1) are close to unity for all the cases. Overall, it may be concluded that the tubular pinch effect in the flowing Chlorella suspension at upstream $Re = 47$ is not sharp enough to lend itself to separation by branched tubes, since the maximum predicted concentration enhancement is less than 8%.

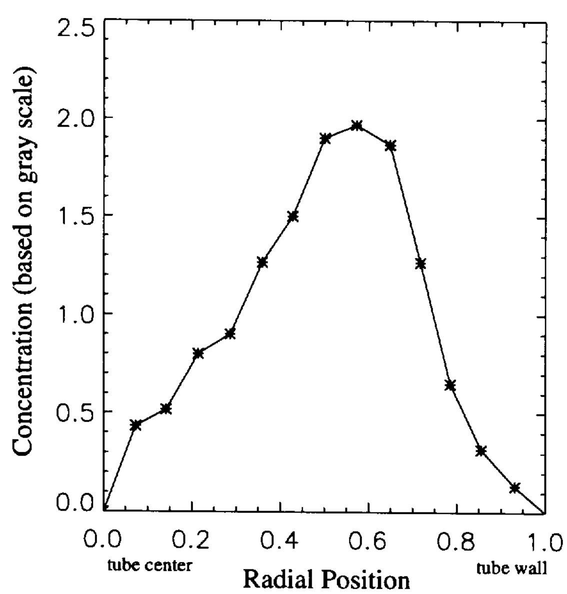

Fig. 8. Upstream concentration profile of Chlorella vulgaris suspension obtained by image analysis; $Re = 47$ and the tube diameter is 203 μm.

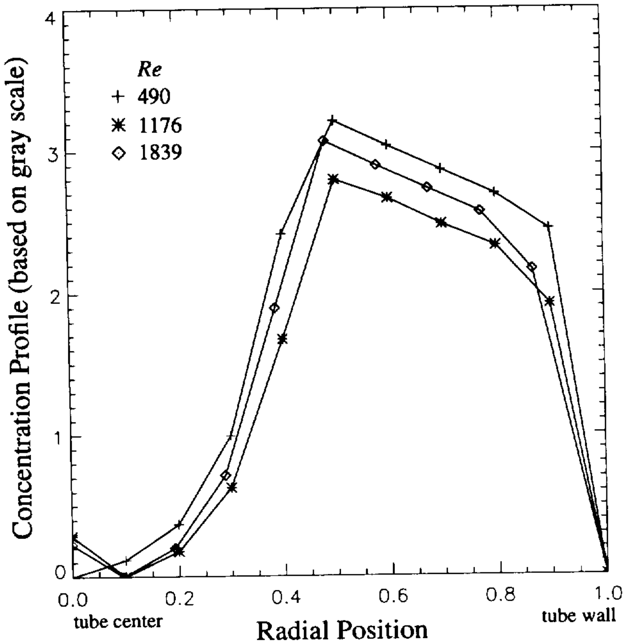

Fig. 9. Upstream concentration profile of Chlorella vulgaris suspension obtained by image analysis; Reynolds numbers are 490, 1176, and 1839, and the tube diameter is 405 μm.

In order to compute the concentration factors that may be obtained at high Reynolds number, the location of the dividing stream surfaces is needed. Dividing stream surfaces were numerically determined for Reynolds number up to 250 in this work. As shown in the section "Dividing stream surfaces in branched tubes", closed dividing stream surfaces are obtained for $Re = 150$ and 250 in two-branch systems. Moreover, it was shown that the circular dividing stream surfaces obtained on the basis of mass balance and the assumption of Poiseuille velocity profile at the tube junction is a very good approximation of the actual dividing stream surfaces at such high Reynolds numbers. Since dividing stream surface locations have not been computed for $Re > 250$ in this work, closed circular dividing stream surfaces will be assumed in order to determine the concentration factors for the high Reynolds number flows. Using the circular dividing stream surfaces and the concentration profiles shown in Fig. 9 the concentration factors at the exits of the branch and the main tube are calculated and shown in Fig. 10 for the two-branch system.

Branch concentration factors as high as 1.3 are obtained for $Re = 1176$. Also, it is observed that the dependence of concentration factors on the flow fraction through the branch for the different high Reynolds numbers in Fig. 10 are quite similar to one another. The peak values of the branch concentration factors for the various Reynolds numbers are close to each other and these peak values are obtained for flow fractions in the range 0.45–0.75. The concentration factors at the exit of the main tube are always less than unity for the flow fraction range shown. This result is expected because the equilibrium radial positions of the spherical Chlorella cells shift toward the tube wall for high Reynolds number, as seen in the particle concentration profiles shown in Fig. 9.

Table 1
Computed concentration factors for Chlorella suspension in T-type bifurcation with two branches at $Re = 47$

Diameter ratio	Flow fraction	Concentration factors	
γ	Q_f	$\langle c_{m,e} \rangle$	$\langle c_b \rangle$
0.5	0.1	1.056	0.496
0.5	0.3	1.03	0.929
1.0	0.1	1.0737	0.330
1.0	0.3	1.0782	0.8175
1.0	0.5	0.985	1.0148

Discussion

In order to achieve separation in a branched tube device, the concentration factor should be significantly greater than 1.0. As seen in the previous section, the maximum concentration factor for Chlorella suspensions is 1.3. For Spirulina platensis suspensions concentration factors as high as 3.0 were obtained in branched tube experiments by Rakow and Fernald (1991). Spirulina mircroalgae particles, which are helical in shape, occupy radial equilibrium positions closer to the tube axis, in contrast to spherical Chlorella which tend to concentrate closer to the tube wall. The different shapes of

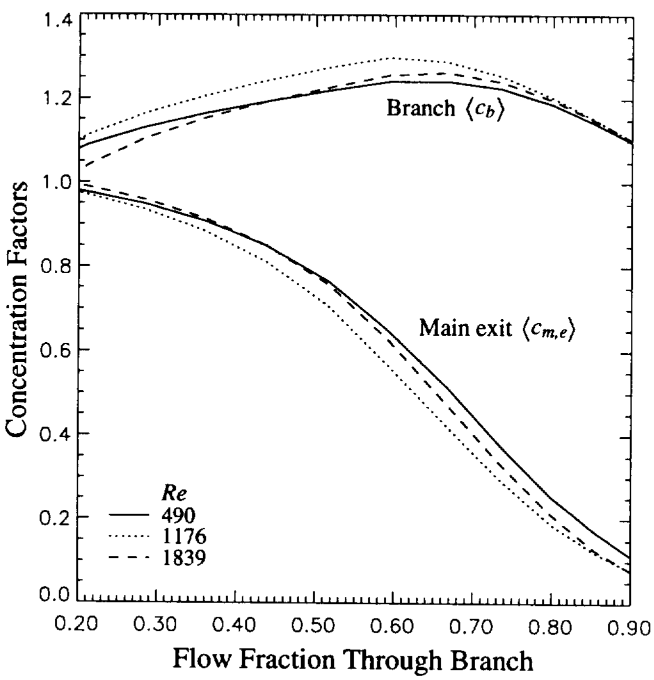

Fig. 10. Predicted concentration factors in two-branch systems for Chlorella vulgaris suspensions as a function of Q_f; Reynolds numbers are 490, 1176, and 1839.

Chlorella and Spirulina is the most likely factor for their distinctly different inertial migration characteristics.

In the experimental observations of Rakow and Fernald (1991), Spirulina suspensions concentrated in the main tube and the dilute part of the flow field exited the branch. A design methodology was developed by Poflee *et al.* (1994) for efficient stage-wise concentration and applied to Spirulina suspensions. Efficient skimming is achieved when the fraction of flow through the branch is manipulated such that the dividing stream surface is located in the dilute region of the flow field. In order to achieve a desired flow fraction through the branch, the branch lengths are varied so as to provide appropriate resistance for the flow splitting (Poflee *et al.*, 1994; Rakow, 1991). A case study was conducted for concentrating Spirulina suspensions from 0.01% *DM* (dry mass fraction) to a final concentration of 1.0% *DM*. It was found that if the design of branched tubes in each stage is based on efficient flow skimming then significantly higher recovery can be obtained as compared to the case when a constant branch tube geometry is used in each stage. In their experiments on concentrating Spirulina suspensions using branched tubes, Ryan *et al.* (1996) obtained an overall concentration factor of 33 using seven stages. They varied the main tube lengths to achieve resistance matching in various stages. Moreover, they obtained an overall recovery of 91.5%.

For the concentration of Chlorella suspensions in branched tubes, however, the predicted stage recovery, f_ψ, at the peak concentration factor is approximately 78% (see Fig. 10 and Eq. *(10)*). At a flow fraction of 0.9 through the branch, where the concentration factor is approximately 1.1, the recovery is 99%. Assuming that these values of concentration factor (1.1) and recovery (99%) are maintained throughout a branched tube stage train designed to achieve a 100-fold increase in concentration (i.e., 0.01% *DM* to 1.0% *DM*) the required number of stages will be 48 with an overall recovery of approximately 60%. In comparison, it is clear that the separation potential for Chlorella using a branched tube device is substantially lower than what has been achieved for Spirulina.

Conclusion

Three-dimensional numerical solutions of Navier-Stokes equations for steady laminar flow of a homogeneous fluid in tubes having one and two orthogonal branches have been obtained. Closed elliptically shaped dividing stream surfaces have been found in two-branch systems for $Re = 150$ and 250, $Q_f = 0.3$, and $\gamma = 0.5$. The maximum predicted concentration factors for Chlorella suspensions in branched tubes is 1.3, which is significantly lower than that observed for suspensions of filamentous Spirulina by Rakow and Fernald (1991). Particle shape may be used as a criterion in selecting suspensions for separation using this method.

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